Homework IV
Due Date: 30/04/2024

Exercise 1. Find the Fourier series of the following functions over its interval.
(i) (1 point) $f(x) = \cos(\frac{x}{2})$, $-\pi \leq x \leq \pi$.
(ii) (1 point) $f(x) = \tan(\frac{x}{2})$, $-\pi \leq x \leq \pi$.
(iii) (1 point) $f(x) = e^x$, $-\pi \leq x \leq \pi$.

Exercise 2. Find the Fourier series of the following functions over its interval.
(i) (1 point) $f(x) = \begin{cases} 0, & -\pi \leq x < 0, \\ 1, & 0 \leq x \leq \pi. \end{cases}$
(ii) (1 point) $f(x) = \begin{cases} x + \pi, & -\pi \leq x < 0, \\ -x + \pi, & 0 \leq x \leq \pi. \end{cases}$
(iii) (1 point) $f(x) = \begin{cases} x + \pi, & -\pi \leq x < 0, \\ x - \pi, & 0 \leq x \leq \pi. \end{cases}$

Exercise 3. Let $f(x) = x^2$, $-\pi \leq x \leq \pi$.
(i) (1 point) Find the Fourier series of $f(x)$.
(ii) (1 point) Use the Pointwise Convergence Theorem for Fouier series to find the value of $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

Exercise 4. Let $f(x) = x$, $-\pi \leq x \leq \pi$.
(i) (1 point) Find the Fourier series of $f(x)$.
(ii) (1 point) Use the Parseval's identity to find the value of $\sum_{n=1}^{\infty} \frac{1}{n^2}$.